Doppler-Shift Measurements of Axial and Rotational Velocities in an MPD Arc

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An MPD arc using a bariated tungsten cathode with NH_3 as a propellant was studied (current: 100–700 amp; magnetic field: 600–5000 gauss; mass flow: 10 and 17.5 mg/sec). A 3.4-m spectrograph (Ebert mounting) was used. Reference lines from an iron arc were used rather than from a side-on view of the plasma beam itself. The shifts were obtained from microdensitometer traces of photographic plates. All ion lines are noticeably slanted with respect to the reference lines indicating strong rotation of the plasma beam. Rotational and axial velocities of nitrogen ions were obtained at different axial positions. Neutral species (nitrogen, hydrogen) show much smaller axial velocities and no rotation. Clear evidence of ion acceleration in the magnetic nozzle downstream of the electrode gap could be established.

Nomenclature

v = velocity of moving ions, atoms
c = speed of light
λ = wavelength
$\Delta\lambda$ = Doppler shift
α = angle between axis of accelerator and optical axis of spectrograph
ω = angular frequency
R = $\lambda/\delta\lambda = n \cdot M$ resolving power of grating
$\delta\lambda$ = smallest detectable wavelength difference
M = total number of grooves on grating
n = spectral order
v_{crit} = critical velocity
E_{ion} = ionization energy
m = mass of ions, atoms
E_{diss} = dissociation energy

Introduction

IN the initial development of the MPD arc, a large effort was made to determine the exhaust velocity (or specific impulse) and the efficiencies from measurements of thrust and mass flow. The accuracy of such measurements, however, is questionable in view of the possibility of entrainment of ambient gas from the vacuum chamber and of mass eroded from the electrodes. Velocities have also been obtained in the exhaust of the MPD arc based on the propagation of natural or artificially introduced fluctuations, directional Langmuir probes and $v \times B$ probes at the Langley Research Center and several other laboratories. Some doubts remain concerning the accuracy of these measurements, partly because the flow is disturbed and partly because uncertainties arise whether the velocities measured were truly the flow velocities.

As a result of these difficulties, considerable interest arose recently in the use of spectroscopic velocity measurements using the Doppler shift of spectral lines. These techniques have the great advantage that they measure the true flow velocities of ions and neutral atoms as long as other effects resulting in line shifts can be disregarded.

For the low velocities obtained in high background pressure MPD arcs, Bohn[1] used an interferometer in connection with a 2-m spectrograph. Sovie[2] reported measurements of axial velocities obtained with a 0.5-m monochromator and a special chopping technique. Additional spectroscopic velocity measurements in MPD arcs have been published recently by Malliaris[3] (AVCO) and Kling[4] (Institut für Plasmadynamik, Stuttgart). While Ref. 4 contains only measurements of ion velocities (A II), Ref. 3 finds that neutral species are much slower and their motion is uncoupled from the acceleration of the ions.

Measuring Technique

The velocities encountered in the exhaust of MPD arcs operated at low background pressure are high enough to cause Doppler shifts which can be detected directly with a high resolution spectrograph. Fortunately, the excited atoms and ions of this low-density plasma emit spectral lines of rather narrow line width (typically 0.2 Å). Unlike many other spectroscopic techniques, the Doppler-shift measurement does not depend on the thermodynamic state of the plasma, that is, the plasma is not required to be in local thermodynamical equilibrium, as long as it can be regarded as optically thin. This restriction means only that one has to make sure that the radiation coming from inner regions of the plasma is not absorbed in outer layers. Of course, one has to be careful not to pick a line which shows self-absorption.

A light source moving at the velocity v with respect to an observer will exhibit slightly shifted spectral lines. This Doppler shift $\Delta\lambda$ is given by

$$\Delta\lambda/\lambda \approx v/c \tag{1}$$

λ = wavelength of unshifted line; c = speed of light. The line will be shifted toward shorter wavelengths if the source is moving toward the observer and to longer wavelengths if it is moving away from the observer.

Assuming the optical axis of the spectrograph to be at an angle $\alpha = 45°$ with respect to the moving plasma, a spectral line of about 5000 Å and velocities of about 10^4 cm/sec, the expected line shift will be given by

$$\Delta\lambda = \lambda \frac{v \cos\alpha}{c} = 5000 \frac{10^6 \times 0.707}{3 \times 10^{10}} \approx 0.1\text{Å} \tag{2}$$

To measure such a small shift within an uncertainty of 20%, it would be desirable to have a resolving power R of about 250,000.

For these measurements, a grating with 160,000 grooves was used which was blazed for 1μ. The observations were

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Table 1 Used N II lines and closest reference lines

N II	Fe	$\lambda_{N\,II} - \lambda_{Fe}$
4643.085	4643.475	−0.390 Å
4630.543	4630.128	0.415 Å
4613.866	4613.219	0.647 Å
4607.157	4607.654	−0.497 Å
4601.480	4602.010	−0.530 Å

made in second order. It was checked that the resolution is about 0.03 Å or better.

A schematic of the experimental arrangement is shown in Fig. 1. Since it could not be excluded that the plasma has small intensity variations, photographic plates were used rather than scanning the spectrum and recording the intensity with phototubes. All scanning techniques are very sensitive to variations in light intensity during the wavelength scan which would seriously influence the accuracy of a wavelength determination.

One major problem is to obtain suitable reference lines. So far it has been standard practice either to use a reference discharge (hollow cathode, high-frequency discharge) or to look in a direction perpendicular to the plasma beam to obtain the unshifted line. The second method produces a rather broad reference line which, in addition, is slanted if the plasma is rotating. The first method yields sharp lines. Both methods have the disadvantage that, because of the minuteness of the shift which is smaller than the width of the plasma line, the reference line and the shifted line will overlap and have to be separated by using a mask in front of the entrance slit. Sovie[2] and Malliaris[3] have recently suggested chopping techniques which are capable of scanning and separating two lines which will otherwise overlap. In connection with a high-resolution spectrograph, these methods should give accurate results provided the intensity variations during the scan are less than a few percent.

In these experiments, the difficulty of overlapping lines was overcome by using a reference light source which produced sharp spectral lines close enough to make an accurate measurement but separated far enough in wavelength not to overlap. The spectrun of an iron arc provided reference lines for five N II lines typically within 0.5 Å. A special argon-stabilized iron arc after Tonner[5] and P. Wulff was built which burned quietly and produced the reference lines in suitable intensities. The reference spectrum was superimposed on the spectrum taken of the moving plasma. Each line pair (N II line and reference line) was scanned at 5 equidistant positions on a microdensitometer. The velocities were calculated from the measured line distances. This method is limited to experimental conditions in which the Doppler effect is the only reason for a line shift. Shifts due to Stark effect can be neglected because of the relatively low electron densities (typically about 10^{14} cm^{-3}) encountered in MPD flows. An experimental check of the absence of other line shift mechanisms is always provided if the viewing direction perpendicular to the plasma motion yields zero shift. (See Table 1.)

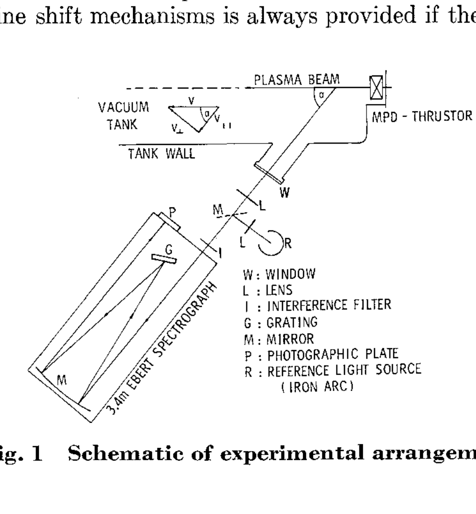

Fig. 1 Schematic of experimental arrangement.

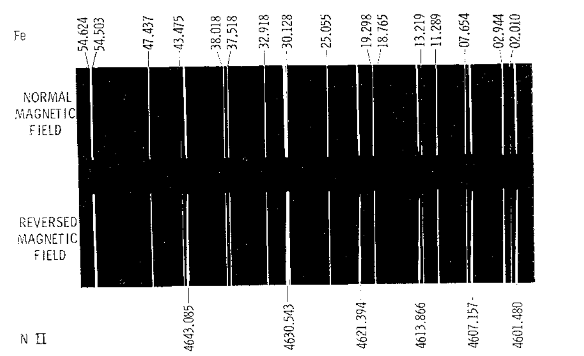

Fig. 2 Spectra showing 6 plasma lines (N II) and superimposed reference lines (Fe) for 2 different directions of applied magnetic field.

The wavelengths are taken from Refs. 6 and 7. Such a spectrum is shown in Fig. 2. It is evident that all plasma lines are slanted with respect to the reference lines. This effect of slanted lines has been observed before in radiation from plasmas in strong external magnetic fields and has been interpreted as being indicative of plasma rotation.[8] Figure 2 shows two spectra taken at normal and reversed magnetic field directions. It is quite noticeable that the plasma lines in the lower spectrum are slanted in the other direction.

Figure 3 shows schematically how a rotating plasma will produce slanted lines. In addition to the shift produced by the translational motion of the plasma (if it has a component in the direction of the observer), one end of the spectral line corresponding to plasma rotating toward the observer will be shifted to shorter wavelengths while the opposite end of the line will be shifted toward longer wavelengths corresponding to plasma moving away from the observer. An observer looking at an angle α to the plasma beam will observe components of translational and rotational velocities. The integral shift recorded on the photographic plate will result from light emitted by volume elements inside a solid double cone, as indicated in Fig. 3. In an optically thin plasma the radiation of a volume element will be proportional to the emission coefficient at a particular wavelength and to the geometrical depth of the volume element. Thus, regions with greater intensity or size will contribute more to the integral shift than neighboring volume elements. It has been shown that the observed integral shift can be unfolded into local shifts under certain assumptions. Assuming axial symmetry and making certain assumptions about the line shape, this problem can be approximated by the well-known Abel inversion formula according to Bohn[1] if the product of intensity and shift is regarded as the quantity to be transformed. Drawin[9] and Ahlborn[10] proposed similar mathe-

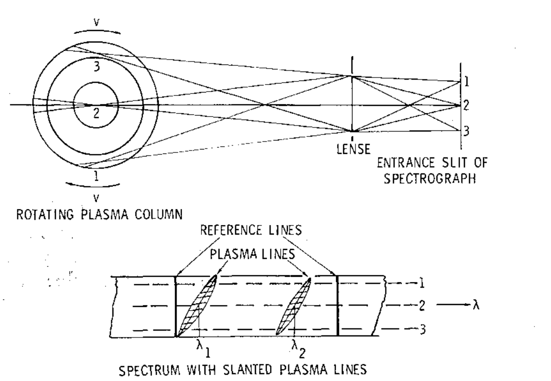

Fig. 3 Schematic indicating how a rotating plasma column will produce slanted spectral lines.

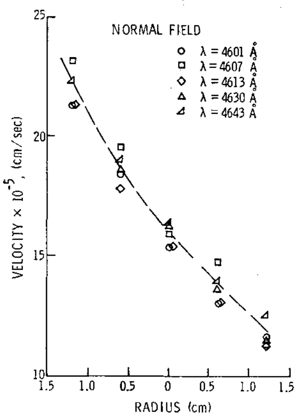

Fig. 4 Velocities obtained from 5 different N II lines at normal magnetic field (viewing angle: $\alpha = 49°$).

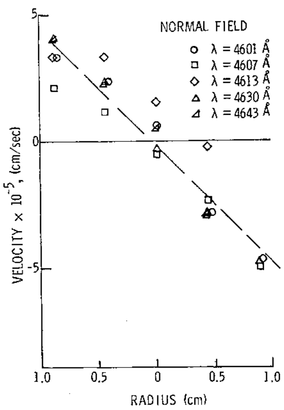

Fig. 6 Velocities obtained from 5 different N II lines at normal magnetic field (viewing angle: $\alpha = 90°$).

matical procedures for obtaining local Doppler shifts from slanted spectral lines.

However, like the Abel inversion, these mathematical unfolding procedures are very sensitive to scatter in the data points and asymmetries in the experimental curves. They are justified only for very reproducible axisymmetric discharges. In the case of the MPD arc, where a rotating disturbance (current spoke) may exist, the value of such unfolding procedures becomes questionable. In these experiments only the integral shift was obtained bearing in mind that it is a weighted average along the line of sight and that it is a time average over the exposure time (between 5 sec and several min). At three positions in the arc, the measurement should be fairly accurate despite the averaging. Points 1 and 3 (Fig. 3) do not require an Abel inversion because the radiation does not have to pass through outer layers. Point 2 should yield the translational velocity because all rotational velocity components are perpendicular to the line of sight.

To obtain a test of the accuracy of this measuring technique, 3 checks were performed: 1) Velocities obtained from five different ion lines of the same spectrum agree reasonably well ($\pm 10^5$ cm/sec). 2) The direction of the magnetic field was reversed. Figures 4 and 5 show that the same translational velocity is obtained for the center of the beam and that the rotational velocities are the same, while the direction of the rotation is reversed. 3) Spectra were taken, looking perpendicular to the plasma beam. Figures 6 and 7 show that the measured translational velocity is zero because it has no component in the direction of the observer.

MPD Arc Design and Operation Conditions

Figure 8 shows a schematic of the electrode design. This MPD arc was designed and used by P. Brockman and has been described in earlier publications (e.g., Ref. 11). The propellant used in these experiments was ammonia. The background pressure under running conditions was always lower than 5×10^{-4} torr. A cathode made of bariated tungsten was used which exhibited much smoother running conditions than the thoriated tungsten cathodes. However, after an extended period of operation (about 50-hr actual operating time) the running became rough and erratic, anode erosion increased considerably, and the monitored arc voltage was higher. It is assumed that the change is due to loss of barium from the cathode which then exhibits the inferior characteristics of pure tungsten. Even at this stage the cathode shows no visible traces of erosion.

The arc current was varied between 100 and 700 amp, the magnetic field between 600 and 5000 gauss (at the cathode tip). Measurements were made at two mass flows: 10 mg/sec and 17.5 mg/sec; and four different axial positions: 2.5-cm, 7-cm, 9.5-cm, and 15-cm downstream.

Experimental Results

Most measurements were taken at a distance of 9.5-cm downstream from the anode face. Figure 9 shows that for the wide range of currents and magnetic fields the axial velocities of nitrogen ions were in the range of (15 ± 1.5) 10^5 cm/sec. No clear indication of increase of axial velocities with input power could be found within the experimental errors. However, the axial velocity increases with distance downstream and is still going up at 15-cm downstream (Fig. 10). The measurements indicate that the rotational velocities of the inner core ($\sim$ a diameter of 2.5 cm) could be approximated by a solid-body rotation $v_{\text{rot}} \approx \omega \cdot r$. The rotational velocity v_{rot} (and the angular frequency ω) was also independent of arc current but showed a monotone increase with magnetic field strength as indicated in Fig. 11. The rotational velocities were highest close to the anode and decreased downstream. While angular frequencies up to 10^6 sec^{-1} were measured at 2.5 cm, the highest obtained angular frequencies at 15-cm downstream were about 4×10^5 sec^{-1}. The behavior of axial and rotational velocities was almost identical for the two mass flows of 10 mg/sec and 17.5 mg/sec used in the experiments.

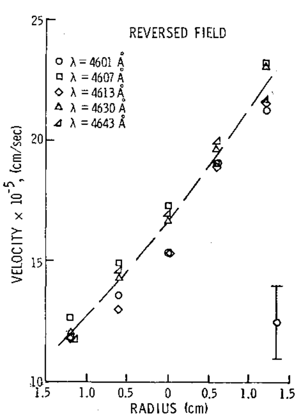

Fig. 5 Velocities obtained from 5 different N II lines at reversed magnetic field (viewing angle: $\alpha = 49°$).

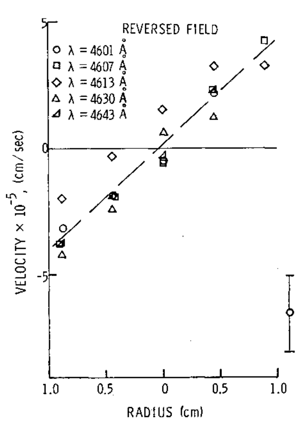

Fig. 7 Velocities obtained from 5 different N II lines at reversed magnetic field (viewing angle: $\alpha = 90°$).

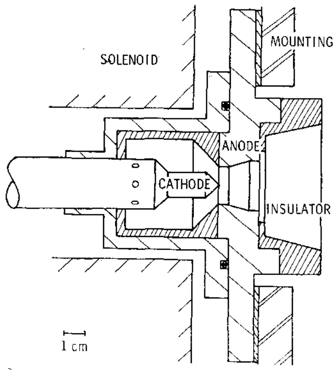

Fig. 8 Schematic of accelerator.

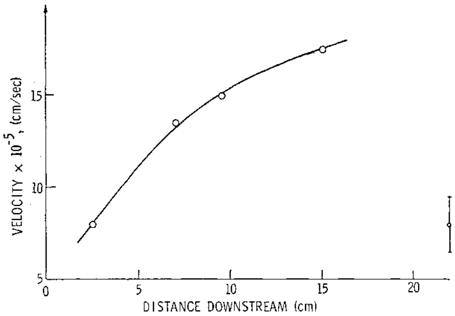

Fig. 10 Axial ion velocity vs distance downstream.

It is of interest to note that an increase of axial velocity with current could be observed when the arc showed irregular fluctuations and occasional outbursts. An increase of axial velocity with current was also obtained in Ref. 2 and in previous measurements at the Langley Research Center. In both experiments thoriated tungsten cathodes and spectrographs with smaller resolving power were used.

In addition to these measurements performed on N II lines, velocities of neutral species at 9.5-cm downstream were obtained. The neutral nitrogen line 4151.46 Å, besides being very dim compared to the ion lines, was very narrow (0.1 Å) and showed no increase of intensity near the center of the plasma beam which may indicate that there are very few neutral particles in the hot core of the plasma beam. The obtained velocities for all conditions were about 3×10^5 cm/sec and there was no indication of rotation. (Angular frequencies $\geq 2 \times 10^5$ sec^{-1} would definitely have been detected.) It is quite conceivable, however, that no information about the center beam is gained from the N I lines because most of the emission might stem from colder outer layers.

Neutral hydrogen particles exhibited a slightly different behavior. The H_α line showed a definite intensity increase near the center core. The measured velocities were in the range of $(4 - 8) \times 10^5$ cm/sec depending on arc current. Again, no rotation could be detected. It should be added that the velocities obtained for neutral hydrogen particles are likely to be less accurate than the other reported velocities. Due to the width of H_α (0.6 Å) the error may be as large as 4×10^4 cm/sec.

Discussion of Experimental Results

The observation of limits to the velocities of the MPD arc independent of the input power suggests evaluation of the results in terms of a critical velocity v_{crit}.Alfvén[12] first proposed a model in which the velocity of a nonionized gas streaming through a plasma in a magnetic field should be limited by a critical velocity

$$v_{crit} = (2E_{ion}/m)^{1/2} \qquad (3)$$

This model was experimentally checked with some success for the homopolar device for conditions where ions rotate through stationary neutrals. Several authors have applied this model to the flow through an MPD arc, whereby the dissociation energy was also included in the definition:

$$\tfrac{1}{2}mv_{crit}^2 = E_{diss} + E_{ion} \qquad (4)$$

Since this critical velocity model assumes an uncoupled relative motion of the ions and neutral particles, it need not apply for all operating conditions of the MPD arc. The neutral particles observed in these experiments move with considerably lower velocities than the ions and thus, it would appear that the critical velocity model would apply. Because of the lack of local resolution, the measurements are, however, not sufficiently complete to prove that the neutrals whose velocities are measured actually belong to the same part of the plasma jet where the ion velocities are recorded.

A numerical estimate of v_{crit} for the nitrogen ions is made by using a dissociation energy of 10–11 ev for producing a nitrogen atom from ammonia (see Ref. 13) and adding to it the ionization energy for nitrogen of 14.5 ev. This yields a critical velocity of about 1.8×10^6 cm/sec for the nitrogen ions. The questions may arise concerning the role played by hydrogen which is also produced in the dissociation of ammonia. Since the ionization energies of hydrogen and nitrogen do not differ much, the presence of hydrogen should not influence v_{crit} of the nitrogen ions very much.

The measured velocities (including axial and rotational components) exceed this critical velocity considerably. This may be due to the fact either that the gas is fully ionized, in which case the model would no longer apply, or that the ions are still accelerated after they leave the region in which ionization takes place. The fact that under all running conditions the mass flow was below or slightly above the critical mass flow and that the neutral nitrogen lines were very weak also indicate a high degree of ionization at these low mass flows. It is also of interest that some velocities of the nitrogen ions correspond to energies of the order of the total applied voltage which should be larger than the available voltage (applied voltage minus sheath drops). The indications that the plasma may be highly ionized permit a rough estimate of the thrust. Assuming that the complete mass flow is accelerated to the measured ion velocities, a thrust of about 20 g force would be expected. This is of the

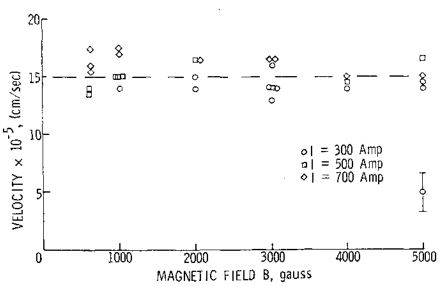

Fig. 9 Axial ion velocity at 9.5-cm downstream vs magnetic field strength.

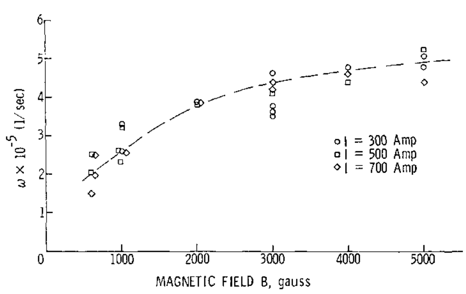

Fig. 11 Angular frequency of plasma rotation at 9.5-cm downstream vs magnetic field strength.

same order of magnitude as thrusts obtained from direct thrust measurements made earlier at this laboratory.

It could be established beyond doubt that the axial velocities increase for an appreciable distance outside the electrode gap. There exists thus strong evidence for acceleration of ions in a so-called magnetic nozzle. Several mechanisms have been proposed for this acceleration involving Hall currents, conversion of rotation to axial motion, and electrothermal expansion.

Evidence for conversion of rotation to axial motion is indicated by a decrease of rotational velocities accompanying a downstream increase of axial velocity. The measurements, however, indicate also an increase of rotational velocities with magnetic field without a corresponding increase in axial velocities. This again suggests that rotation may not be efficiently converted to axial motion and that part of the rotation may be used to heat the plasma by dissipation; the latter may also play some role in decreasing the rotation in the downstream direction.

An axial acceleration involving Hall currents could reach a limit with increasing ion rotation and thus could perhaps explain a limit in axial velocity. However, the measurements are not sufficiently complete to determine the relative contributions of rotational conversion, Hall current, and electrothermal acceleration to the acceleration mechanism. It is of considerable interest that such high unconverted rotational velocities have been observed at large axial distances from the electrodes, resulting in very high total velocities (axial plus rotational). These high rotational velocities may be related to the presence of $j \times B$ volume forces due to bulging currents. One hopes that with better choice of electrodes, magnetic field shape, and propellant, the axial velocities may perhaps be further increased at the expense of rotational energy.

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